Holst action and Dynamical Electroweak symmetry breaking

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Abstract

We consider Poincare gravity coupled in a nonminimal way to spinors. The gravitational action is considered that contains both Palatini and Holst terms. Due to torsion the effective four - fermion interactions appear that may lead to the left - right asymmetry and the condensation of fermions. When the mass parameter entering the mentioned terms of the gravitational action is at a Tev scale the given construction may provide the dynamical Electroweak symmetry breaking. This is achieved via an arrangement of all Standard Model fermions in the left - handed Dirac spinors while the right - handed spinors are reserved for the technifermions. Due to the gravitational action the technifermions are condensed and, therefore, cause the appearance of gauge boson masses.

1 Introduction

Holst action [1, 2, 3, 4, 5] breaks parity. Therefore, when Poincare gravity that contains Holst term in the action is coupled to fermions, it may provide the appearance of parity breaking four - fermion interactions [6, 7, 8, 9]. The four - fermion interactions induced by Holst action were considered in [10, 11, 12] as a source of fermion condensation used, mainly, in a cosmological background. In our previous paper [13] we have suggested that nonminimal coupling of fermion fields to torsion [14] may provide condensation of the additional fermions (called technifermions in an analogy with Technicolor theory (TC) [15, 16, 17]) and provide Dynamical Electroweak Symmetry Breaking (DEWSB) if torsion mass parameter is at a Tev scale.

Namely, we arrange all SM fermions in the left - handed components of the Dirac spinors while the additional fermions (called technifermions) are

arranged in right - handed components of the spinors. If the parity breaking is admitted in the torsion action, under natural assumptions this action has the form that leads to appearance of the asymmetry between the left-handed and the right-handed fermions. Due to this asymmetry the four fermion interactions provide condensation of the technifermions while do not affect qualitatively the dynamics of the SM particles. As a result the Electroweak symmetry is broken.

In the suggested approach the problems specific for TC, Extended Technicolor (ETC) [18, 19, 20, 21, 22, 23, 24, 25, 26], and Bosonic Technicolor [27, 28, 29] are avoided. However, the suggested approach contains its own difficulties. First, the source of the parity violating action for torsion was not suggested. Second, the effective theory that contains four - fermion interactions is to be treated in a way similar to the effective Nambu - Jona - Lasinio model (NJL) of chiral symmetry breaking [30] in TC. Namely, the NJL model is a nonrenormalizable finite cutoff theory. And physical results depend on the value of the cutoff Λ_χ that becomes an additional physical parameter.

In the present paper we suggest a possible origin of the parity violating action. Namely, in Poincare gravity the torsion field is to manifest itself through the curvature of Riemann - Cartan space. There are only two possible terms constructed of curvature that contain squared torsion. These two terms are Palatini and Holst terms. We consider both these terms. In addition we consider the nonminimal coupling of spinors to gravity that itself admits parity violation. We consider several limiting cases and derive the conditions under which the resulting four - fermion interactions are repulsive for SM fermions and attractive for the technifermions (or the induced interactions for SM fermions are negligible compared to that of the technifermions). In all these cases the theory admits condensation of technifermions while SM fermions remain massless. That's why we partially resolve the first difficulty mentioned above. As for the second difficulty, we cannot resolve it at the present moment and consider it as a subject of future investigations.

The paper is organized as follows. In the 2-nd section we consider fermion fields in Riemann-Cartan space. In the 3-rd section we consider Holst action and derive the four - fermion interactions that appear after integration over torsion. In the 4 - th section we consider several limiting cases and point out the regions in space of couplings, where right-handed fermions may be condensed while left - handed fermions remain massless. In the 5 - th section we introduce two kinds of spinors nonminimally coupled to gravity. The left

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- handed components of these spinors are used to arrange both left - handed and right - handed SM fermions while the right - handed components of these spinors are used to arrange technifermions. We demonstrate how the resulting four - fermion action can be written in terms of 4 - component SM fermions and technifermions. In the 6 - th section we apply NJL technique to the four - fermion interactions of technifermions and describe how chiral symmetry breaking occurs. In section 7 we introduce mass term for original spinors that contain SM fermions as their left - handed components. We then derive the mass term for the SM fermions. In section 8 we end with the conclusions.

2   Fermions in Riemann-Cartan space

We consider the action of a massless Dirac spinor in Riemann - Cartan space in the form [9]:

$$S_f \;\; = \;\; \frac{i}{2} \int E\{\bar\psi\gamma^\mu(\zeta - i\chi\gamma^5)D_\mu\psi - [D_\mu\bar\psi](\bar\zeta - i\bar\chi\gamma^5)\gamma^\mu\psi\}d^4x \qquad (1)$$

Here $\zeta = \eta + i\theta$ and $\chi = \rho + i\tau$ are the coupling constants, $E = \det E_\mu^a$, E_μ^a is the inverse vierbein, $\gamma^\mu = E_a^\mu\gamma^a$, the covariant derivative is $D_\mu = \partial_\mu + \frac{1}{4}(\omega_\mu^{ab} + C_\mu^{ab})\gamma_{[a}\gamma_{b]}$; $\gamma_{[a}\gamma_{b]} = \frac{1}{2}(\gamma_a\gamma_b - \gamma_b\gamma_a)$. The torsion free spin connection is denoted by ω_μ while C_μ is the contorsion tensor. They are related to E_μ^a, Affine connection Γ_{jk}^i, and torsion $T_{.\mu\nu}^a = T_{.\mu\nu}^\rho E_\rho^a$ as follows:

$$\nabla_\nu E_\mu^a \;\; = \;\; \partial_\nu E_\mu^a - \Gamma_{\mu\nu}^\rho E_\rho^a + \omega_{.b\nu}^a E_\mu^b + C_{.b\nu}^a E_\mu^b = 0$$
$$\tilde D_{[\nu} E_{\mu]}^a \;\; = \;\; \partial_{[\nu} E_{\mu]}^a + \omega_{.b[\nu}^a E_{\mu]}^b = 0$$
$$T_{.\mu\nu}^a \;\; = \;\; D_{[\nu} E_{\mu]}^a = \partial_{[\nu} E_{\mu]}^a + \omega_{.b[\nu}^a E_{\mu]}^b + C_{.b[\nu}^a E_{\mu]}^b = C_{.b[\nu}^a E_{\mu]}^b \qquad (2)$$

This results in:

$$\Gamma_{\mu\nu}^\rho \;\; = \;\; \{^\rho_{\mu\nu}\} + C_{.\mu\nu}^\rho$$
$$C_{.\mu\nu}^\rho \;\; = \;\; \frac{1}{2}(T_{.\mu\nu}^\rho - T_{\nu.\mu}^{.\rho} + T_{\mu\nu}^{..\rho})$$
$$\{^\alpha_{\beta\gamma}\} \;\; = \;\; \frac{1}{2}g^{\alpha\lambda}(\partial_\beta g_{\lambda\gamma} + \partial_\gamma g_{\lambda\beta} - \partial_\lambda g_{\beta\gamma})$$
$$\omega_{ab\mu} \;\; = \;\; \frac{1}{2}(c_{abc} - c_{cab} + c_{bca})E_\mu^c$$

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$$C_{ab\mu} = \frac{1}{2}(T_{abc} - T_{cab} + T_{bca})E^c_\mu \qquad (3)$$

Here $c_{abc} = \eta_{ad}E^\mu_b E^\nu_c \partial_{[\nu}E^d_{\mu]}$; $T_{abc} = \eta_{ad}E^\mu_b E^\nu_c T^d_{.\mu\nu}$; $g_{\mu\nu} = E^a_\mu E^b_\nu \eta_{ab}$; $\Gamma^\rho_{\mu\nu} - \Gamma^\rho_{\nu\mu} = T^\rho_{.\mu\nu}$; indices are lowered and lifted via g and E as usual.

(1) can be rewritten as follows:

$$\begin{aligned}
S_f &= \frac{1}{2}\int E\{i\bar\psi\gamma^\mu(\zeta - i\chi\gamma^5)\tilde D_\mu\psi - i[\tilde D_\mu\bar\psi](\bar\zeta - i\bar\chi\gamma^5)\gamma^\mu\psi \\
&\quad + \frac{i}{4}C_{abc}\bar\psi[\{\gamma^{[a}\gamma^{b]},\gamma^c\}(\eta + \tau\gamma^5) - i[\gamma^{[a}\gamma^{b]},\gamma^c](\theta - \rho\gamma^5)]\psi\}d^4x \quad (4)
\end{aligned}$$

Here $\tilde D$ is the covariant derivative of general relativity. Next, we obtain:

$$\begin{aligned}
S_f &= \frac{1}{2}\int E\{i\bar\psi\gamma^\mu(\zeta - i\chi\gamma^5)\tilde D_\mu\psi - i[\tilde D_\mu\bar\psi](\bar\zeta - i\bar\chi\gamma^5)\gamma^\mu\psi \\
&\quad -\frac{1}{4}C_{abc}\bar\psi[-2\epsilon^{abcd}\gamma^5\gamma_d(\eta + \tau\gamma^5) + 4\eta^{c[a}\gamma^{b]}(\theta - \rho\gamma^5)]\psi\}d^4x \quad (5)
\end{aligned}$$

Now let us introduce the irreducible components of torsion:

$$\begin{aligned}
S^i &= \epsilon^{jkli}T_{jkl} \\
T_i &= T^j_{.ij} \\
T_{ijk} &= \frac{1}{3}(T_j\eta_{ik} - T_k\eta_{ij}) - \frac{1}{6}\epsilon_{ijkl}S^l + q_{ijk} \qquad (6)
\end{aligned}$$

In terms of S and T (26) can be rewritten as:

$$\begin{aligned}
S_f &= \frac{1}{2}\int E\{i\bar\psi\gamma^\mu(\zeta - i\chi\gamma^5)\tilde D_\mu\psi - i[\tilde D_\mu\bar\psi](\bar\zeta - i\bar\chi\gamma^5)\gamma^\mu\psi \\
&\quad +\frac{1}{4}\bar\psi[\gamma^5\gamma_d(\eta S^d - 4\rho T^d) - (\tau S^b + 4\theta T^b)\gamma_b]\psi\}d^4x \quad (7)
\end{aligned}$$

3 Holst action and Dirac fermions

Let us consider the Holst action:

$$S_T = -M_T^2\int EE^\mu_a E^\nu_b G^{ab}_{\mu\nu}d^4x - \frac{M_T^2}{\gamma}\int EE^\mu_a E^{\nu*}_b G^{ab}_{\mu\nu}d^4x \qquad (8)$$

Here $G^{ab}_{\mu\nu} = [D_\mu, D_\nu]$ is the $SO(3,1)$ curvature of Riemann-Cartan space while $^*G^{ab}_{\mu\nu} = \frac{1}{2}\epsilon^{ab}_{..cd}G^{cd}_{\mu\nu}$ is its dual tensor. In the absence of torsion the second

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term is the integral of a total derivative and, therefore, disappears from classical consideration. However, in presence of torsion, it gives nontrivial part to the fermion interactions as will be seen later.

Now let us represent Holst action in terms of torsion and Riemannian curvature [8]:

$$S_T = M_T^2 \int E\{-R + \frac{2}{3}T^2 - \frac{1}{24}S^2 + \frac{1}{3\gamma}TS\}d^4x + \tilde{S} \qquad (9)$$

Here R is Riemannian scalar curvature, $\tilde{S}$ contains the terms that depend on q and the so-called Nieh - Yan invariant.

Let us now suppose for a moment that there are no other terms that depend on torsion in the gravitational action. Then integration over torsion degrees of freedom can be performed for the system that consists of the Dirac fermion coupled to gravity. The result of this integration is:

$$
\begin{aligned}
S_f &= \frac{1}{2}\int E\{i\bar{\psi}\gamma^\mu(\zeta - i\chi\gamma^5)\tilde{D}_\mu\psi - i[\tilde{D}_\mu\bar{\psi}](\bar{\zeta} - i\bar{\chi}\gamma^5)\gamma^\mu\psi\}d^4x \\
&\quad - \frac{3\gamma^2}{(1+\gamma^2)32M_T^2}\int E\{V^2[\theta^2 - \tau^2 + \frac{2\theta\tau}{\gamma}] + A^2[\rho^2 - \eta^2 - \frac{2\eta\rho}{\gamma}] \\
&\quad + 2AV[\theta\rho + \tau\eta + \frac{\rho\tau - \theta\eta}{\gamma}]\}d^4x + S_{eff}[E] \qquad (10)
\end{aligned}
$$

Here we have defined:

$$
\begin{aligned}
V_\mu &= \bar{\psi}\gamma_\mu\psi \\
A_\mu &= \bar{\psi}\gamma^5\gamma_\mu\psi
\end{aligned}
\qquad (11)
$$

S_{eff} is the effective action that depends on metric field only. It comes from the functional determinant after the integration over torsion is performed. If terms that contain derivatives of torsion are absent, S_{eff} is reduced to the renormalization of the cosmological constant. For this reason we omit it below. The four fermion term of (14) differs from that of obtained in [9] by the overall sign and the sign of the immirzi parameter γ due to the difference in the definition of action (8).

Now let us introduce the right-handed and the left-handed currents:

$$
\begin{aligned}
J_+^\mu &= \bar{\psi}_+\gamma^\mu\psi_+ \\
J_-^\mu &= \bar{\psi}_-\gamma^\mu\psi_-
\end{aligned}
\qquad (12)
$$

Here ψ_+ is the right - handed component of ψ while ψ_- is the left - handed component. In the further consideration we consider the case $E^a_\mu = \delta^a_\mu$, and $\omega_\mu = 0$. We also rescale left - handed and right - handed components of ψ:

$$\psi_- \to \frac{1}{\sqrt{\eta + \tau}}\psi_- \ ; \ \psi_+ \to \frac{1}{\sqrt{\eta - \tau}}\psi_+ \tag{13}$$

Now (10) can be rewritten as follows:

$$
\begin{aligned}
S_f \ &= \ \frac{1}{2}\int\{i\bar{\psi}\gamma^\mu\partial_\mu\psi - i[\partial_\mu\bar{\psi}]\gamma^\mu\psi\}d^4x \\
&- \frac{3\gamma^2}{(1+\gamma^2)32M_T^2}\int\{J_+^2[-1+\frac{(\theta+\rho)^2}{(\eta-\tau)^2} - \frac{2(\theta+\rho)}{(\eta-\tau)\gamma}] \\
&+ J_-^2[-1+\frac{(\theta-\rho)^2}{(\eta+\tau)^2} + \frac{2(\theta-\rho)}{(\eta+\tau)\gamma}] \\
&+ 2J_+J_-[1+\frac{\theta^2-\rho^2}{\eta^2-\tau^2} + \frac{\theta+\rho}{(\eta-\tau)\gamma} - \frac{\theta-\rho}{(\eta+\tau)\gamma}]\}d^4x \tag{14}
\end{aligned}
$$

The next step would be to consider the fermions coupled to Poincare gravity with higher derivative action for the gravitational fields. It is well - known that such a theory could be renormalizable in the presence of terms quadratic in curvature [32, 33]. The models of this kind, however, suffer from the so-called unitarity problem. Moreover, the possibility to obtain Newtonian limit is questionable. Nevertheless, below we suppose that a self-consistent Poincare gravity theory exists, probably, with the elements involved that are not known at present. Our main supposition here is that Poincare gravity has two different scales. The first one (around Plamk mass m_p) is related to Riemannian geometry and produces Einstein - Hilbert action in the low energy approximation. The second scale Λ_χ is related to dynamical torsion theory. The effective charge entering the term of the action with the derivative of torsion depends on the ratio ϵ/Λ_χ, where ϵ is the energy scale of the considered physical process. As it will be explained further we expect that Λ_χ is at most one order of magnitude larger than 1 Tev. In addition we also have the mass parameter M_T of (8) that is supposed to be at a Tev scale.

At the scale Λ_χ in addition to (8) the torsional part of the whole gravitational action contains terms that depend on the derivatives of T and S. This means, in particular, that the following terms may enter the action:

$$S_T \ = \ \beta_1\int EG^{abcd}G_{abcd}d^4x + \beta_2\int EG^{abcd}G_{cdab}d^4x$$

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$$+\beta_3 \int EG^{ab}G_{ab}d^4x + \beta_4 \int EG^{ab}G_{ba}d^4x$$

$$+\beta_5 \int EG^2 d^4x + \beta_6 \int EA^{abcd}A_{abcd}d^4x \tag{15}$$

with coupling constants $\beta_{1,2,3,4,5,6}$. Here $G^{abcd} = E^c_\mu E^d_\nu G^{ab}_{\mu\nu}$, $G^{ac} = G^{abc}_{...b}$, $G = G^a_a$, $A_{abcd} = \frac{1}{6}(G_{abcd} + G_{acdb} + G_{adbc} + G_{bcad} + G_{bdca} + G_{cdab})$. Actually, action (15) is the most general quadratic in curvature action that does not contain Parity breaking.

Then the integration over torsion leads to (14) in the low energy limit $\epsilon << \Lambda_\chi$. That's why the obtained theory with the four - fermion action is only the effective low energy approximation that works at the energies much less than the scale Λ_χ of Poincare gravity.

4   Limiting cases

In this section we consider different limiting cases of (14). Our aim is to find out the possibility that there exist attractive interactions between the right - handed fermions while the interaction between the left-handed fermions is either repulsive or is negligible compared to that of the right -handed fermions. We also need the interaction between the right-handed and the left-handed fermions is negligible. All this is needed in order to provide the condensation of right-handed fermions used in the next sections in order to provide DEWSB.

4.1   Einstein - Cartan gravity

This case corresponds to infinite immirzi parameter γ. We have:

$$
\begin{aligned}
S_f &= \frac{1}{2} \int \{i\bar\psi\gamma^\mu\partial_\mu\psi - i[\partial_\mu\bar\psi]\gamma^\mu\psi\}d^4x \\
&\quad -\frac{3}{64M_T^2}\int\{J_+^2[-1 + \frac{(\theta+\rho)^2}{(\eta-\tau)^2}] \\
&\quad +J_-^2[-1 + \frac{(\theta-\rho)^2}{(\eta+\tau)^2}] \\
&\quad +2J_+J_-[1 + \frac{\theta^2-\rho^2}{\eta^2-\tau^2}]\}d^4x
\end{aligned}
\tag{16}
$$

In order for the cross term to vanish we need $1+\frac{\theta^2-\rho^2}{\eta^2-\tau^2}=0$ that is $|\zeta|=|\chi|$. We find repulsive interactions between J_- and attractive interactions between J_+ at

$$|\zeta| = |\chi|$$
$$|\mathrm{Re}\zeta + \mathrm{Im}\chi| < |\mathrm{Im}\zeta - \mathrm{Re}\chi|$$
$$|\mathrm{Re}\zeta - \mathrm{Im}\chi| > |\mathrm{Im}\zeta + \mathrm{Re}\chi| \tag{17}$$

Interactions between J_- disappear if $|\mathrm{Re}\zeta + \mathrm{Im}\chi| = |\mathrm{Im}\zeta - \mathrm{Re}\chi|$.

4.2 Holst term in the action

Let us consider the situation, when Palatini action is absent and only the Holst term is present. We have:

$$
\begin{aligned}
S_f &= \frac{1}{2}\int\{i\bar\psi\gamma^\mu\partial_\mu\psi - i[\partial_\mu\bar\psi]\gamma^\mu\psi\}d^4x \\
&\quad -\frac{3\gamma}{32M_T^2}\int\{J_+^2[-\frac{2(\theta+\rho)}{(\eta-\tau)}] + J_-^2[\frac{2(\theta-\rho)}{(\eta+\tau)}] \\
&\quad +2J_+J_-[\frac{\theta+\rho}{(\eta-\tau)} - \frac{\theta-\rho}{(\eta+\tau)}]\}d^4x
\end{aligned} \tag{18}
$$

In order for the cross term to vanish we need

$$\gamma\frac{\theta+\rho}{(\eta-\tau)} = \gamma\frac{\theta-\rho}{(\eta+\tau)} = \alpha \tag{19}$$

where α is the new coupling constant.

Then we have:

$$
\begin{aligned}
S_f &= \frac{1}{2}\int\{i\bar\psi\gamma^\mu\partial_\mu\psi - i[\partial_\mu\bar\psi]\gamma^\mu\psi\}d^4x \\
&\quad -\frac{3\alpha}{16M_T^2}\int\{J_-^2 - J_+^2\}d^4x
\end{aligned} \tag{20}
$$

We find repulsive interactions between J_- and attractive interactions between J_+ at

$$\gamma\frac{\theta+\rho}{(\eta-\tau)} = \gamma\frac{\theta-\rho}{(\eta+\tau)} = \alpha > 0 \tag{21}$$

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4.3 General case

In general case in order to have attractive interactions between technifermions and repulsive interactions between SM fermions we need:

$$\frac{\gamma^2}{(1+\gamma^2)2}\{1 - \frac{(\theta+\rho)^2}{(\eta-\tau)^2} + \frac{2(\theta+\rho)}{(\eta-\tau)\gamma}\} = \alpha_+ > 0$$

$$\frac{\gamma^2}{(1+\gamma^2)2}\{-1 + \frac{(\theta-\rho)^2}{(\eta+\tau)^2} + \frac{2(\theta-\rho)}{(\eta+\tau)\gamma}\} = \alpha_- > 0 \qquad (22)$$

In order to exclude the cross term $J_+ J_-$ we need

$$|\frac{\gamma^2}{(1+\gamma^2)2}\{1 + \frac{\theta^2-\rho^2}{\eta^2-\tau^2} + \frac{\theta+\rho}{(\eta-\tau)\gamma} - \frac{\theta-\rho}{(\eta+\tau)\gamma}\}| = |\alpha_{+-}| << \alpha_+ \qquad (23)$$

Now let us consider the following region of couplings:

$$|\eta - \tau| << 1$$
$$|\eta + \tau| \sim |\theta - \rho| \sim |\theta + \rho| \sim 1$$
$$\gamma \sim 1 \qquad (24)$$

In this domain we can neglect the terms with J_-^2 and $J_- J_+$, and the action receives the form:

$$S_f = \frac{1}{2}\int\{i\bar\psi\gamma^\mu\partial_\mu\psi - i[\partial_\mu\bar\psi]\gamma^\mu\psi\}d^4x$$
$$- \frac{3\gamma^2}{(1+\gamma^2)32M_T^2}\frac{(\theta+\rho)^2}{(\eta-\tau)^2}\int J_+^2 d^4x \qquad (25)$$

This is a repulsive interaction between the right-handed fermions. In order to obtain attractive interactions one may change the overall sign in (8). Formally this is equivalent to the change $M_T \to iM_T$. It is worth mentioning that this situation corresponds to the sign of Palatini action opposite to the conventional one.

4.4 Discussion

If action (8) is present with finite γ, while the fermion action contains nonzero bare constants ζ and χ, all of the effective coupling constants θ, η, ρ, τ receive contributions from loop corrections due to the dynamical torsion. Of course,

this may destroy the conditions (17), (19), (23), (22) and a kind of fine tuning is necessary to keep the precise (or, almost precise) requirement (23).

A particulary interesting case is when the Palatini action and the bare coupling χ are absent. So, we have the only dimensionless coupling $\alpha = \alpha_+ = \alpha_- = \frac{\theta \gamma}{\eta}$. The necessary condition to be imposed on the terms of the action that depend on the derivatives of torsion is that they do not produce the Palatini action in the low energy limit. This condition can be fulfilled, in particular, if the given terms are conformal invariant. It is worth mentioning here that the usual conformal gravity at a first look contradicts with the Newtonian limit. However, this difficulty may be overcomed, in principle (see, for example, [31]). As for the effective coupling χ, it may appear due to loop corrections because the torsional action breaks Parity. So, if this limiting case is chosen it is necessary to consider renormalization group trajectories for χ in order to find out the domain of the theory, where effective coupling $\chi(\epsilon)$ vanishes. Then we are left with the only requirement $\alpha > 0$.

5 Composite Dirac fields

Below we assume that due to the gravitational action at the considered energies the translational connection E_μ^a is close to δ_μ^a while usual Christoffel symbols vanish. Let us consider two Dirac spinors ψ and ϕ coupled to gravity. Then we consider the fermion action of the form:

$$
\begin{aligned}
S_f &= \frac{1}{2} \int \{ i \bar{\psi} \gamma^\mu (\zeta - i\chi\gamma^5) D_\mu \psi - i [D_\mu \bar{\psi}] (\bar{\zeta} - i\bar{\chi}\gamma^5) \gamma^\mu \psi \} d^4 x \\
&\quad + \frac{1}{2} \int \{ i \bar{\phi}^c \gamma^\mu (\zeta - i\chi\gamma^5) D_\mu \phi^c - i [D_\mu \bar{\phi}^c] (\bar{\zeta} - i\bar{\chi}\gamma^5) \gamma^\mu \phi^c \} d^4 x \quad (26)
\end{aligned}
$$

Here $\phi^c = i\gamma^2 \begin{pmatrix} \phi_- \\ \phi_+ \end{pmatrix}^* = \begin{pmatrix} i\sigma^2 \phi_+^* \\ -i\sigma^2 \phi_-^* \end{pmatrix}$. Below we use the following representation of γ matrices: $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$, where $\bar{\sigma}^0 = \sigma^0 = 1; \bar{\sigma}^i = -\sigma^i \ (i = 1, 2, 3); \gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

According to the previous sections integration over torsion after suitable rescaling of fermion fields leads to

$$
S_f = \int \{ i\psi_+^+ \sigma^\mu \partial_\mu \psi_+ + i\psi_-^+ \bar{\sigma}^\mu \partial_\mu \psi_- + i\phi_+^+ \sigma^\mu \partial_\mu \phi_+ + i\phi_-^+ \bar{\sigma}^\mu \partial_\mu \phi_-
$$

$$+\frac{3\alpha_+}{16M_T^2}(\psi_+^+\sigma^i\psi_+-\phi_-^+\bar\sigma^i\phi_-)^2-\frac{3\alpha_-}{16M_T^2}(\phi_+^+\sigma^i\phi_+-\psi_-^+\bar\sigma^i\psi_-)^2\}d^4x\,(27)$$

Now let us compose new spinors $\psi_t=\begin{pmatrix}\phi_-\\\psi_+\end{pmatrix}$ and $\psi_s=\gamma^5\begin{pmatrix}\psi_-\\\phi_+\end{pmatrix}$. Then we come to the following expression for the effective action:

$$
\begin{aligned}
S_f &= \int\{i\bar\psi_s\gamma^\mu\partial_\mu\psi_s-\frac{3\alpha_-}{16M_T^2}(\bar\psi_s\gamma^i\gamma^5\psi_s)(\bar\psi_s\gamma_i\gamma^5\psi_s)\}d^4x\\
&+\int\{i\bar\psi_t\gamma^\mu\partial_\mu\psi_t+\frac{3\alpha_+}{16M_T^2}(\bar\psi_t\gamma^i\gamma^5\psi_t)(\bar\psi_t\gamma_i\gamma^5\psi_t)\}d^4x
\end{aligned}
\tag{28}
$$

As in the previous section we assume $\alpha_-,\alpha_+>0$. Then the four-fermion interaction for the field ψ_t is attractive while the interaction terms for ψ_s are repulsive. This opens a possibility that ψ_t is condensed while ψ_s is not condensed.

6   Electroweak symmetry breaking

Let us arrange all left - handed fermions and right - handed fermions of the Standard Model in the left - handed parts of Dirac spinors. Correspondingly, the additional fields are arranged within the right-handed parts of the given spinors. We call the mentioned additional fermion fields technifermions. The effective low energy action has the form:

$$
S_f=\int\{i\bar\psi_s^a\gamma^\mu D_\mu\psi_s^a-\frac{3\alpha_-}{16M_T^2}(\bar\psi_s^a\gamma^i\gamma^5\psi_s^a)(\bar\psi_s^b\gamma_i\gamma^5\psi_s^b)\}d^4x
$$
$$
+\int\{i\bar\psi_t^a\gamma^\mu D_\mu\psi_t^a+\frac{3\alpha_+}{16M_T^2}(\bar\psi_t^a\gamma^i\gamma^5\psi_t^a)(\bar\psi_t^b\gamma_i\gamma^5\psi_t^b)\}d^4x
\tag{29}
$$

Here indices a,b enumerate the mentioned Dirac spinors while the derivative D contains all Standard Model gauge fields. Let us apply Fierz transformation to the four fermion term of (29):

$$
\begin{aligned}
S_4 &= \int\{-\frac{3\alpha_-}{16M_T^2}(\bar\psi_s^a\gamma^i\gamma^5\psi_s^a)(\bar\psi_s^b\gamma_i\gamma^5\psi_s^b)\}d^4x\\
&+\int\{\frac{3\alpha_+}{16M_T^2}(\bar\psi_t^a\gamma^i\gamma^5\psi_t^a)(\bar\psi_t^b\gamma_i\gamma^5\psi_t^b)\}d^4x
\end{aligned}
$$

$$
= \frac{3\alpha_+}{16M_T^2} \int \{4(\bar{\psi}_{t,L}^a \psi_{t,R}^b)(\bar{\psi}_{t,R}^b \psi_{t,L}^a)
$$
$$
+ [(\bar{\psi}_{t,L}^a \gamma_i \psi_{t,L}^b)(\bar{\psi}_{t,L}^b \gamma^i \psi_{t,L}^a) + (L \longleftrightarrow R)]\} d^4x
$$
$$
- \frac{3\alpha_-}{16M_T^2} \int \{4(\bar{\psi}_{s,L}^a \psi_{s,R}^b)(\bar{\psi}_{s,R}^b \psi_{s,L}^a)
$$
$$
+ [(\bar{\psi}_{s,L}^a \gamma_i \psi_{s,L}^b)(\bar{\psi}_{s,L}^b \gamma^i \psi_{s,L}^a) + (L \longleftrightarrow R)]\} d^4x \qquad (30)
$$

In this form the action has the form similar to the extended NJL model for ψ_t (see Eq. (4), Eq. (5), Eq. (6) of [30]) (with negative G_V, though). In addition we have the repulsive interactions between ψ_s. In the absence of the Standard Model (SM) gauge fields the $SU(\mathcal{N})_L \otimes SU(\mathcal{N})_R$ symmetry of (29) is broken down to $SU(\mathcal{N})_V$ (here $\mathcal{N}$ is the total number of SM fermions). The SM interactions act as a perturbation.

For the purpose of the further consideration we denote by $N_t = 24$ the number of technifermions; $G_S = \frac{3\alpha_+ N_t \Lambda_\chi^2}{16M_T^2 \pi^2}$; $G_V = -\frac{1}{4}G_S$. Here Λ_χ is the cutoff that is now the physical parameter of the model. Its value depends on the details of physics that provides the appearance of the four - fermion interactions. In our case Λ_χ is to be calculated within the (unknown) Poincare gravity theory. We also denote $g_s = \frac{4\pi^2 G_S}{N_t \Lambda_\chi^2} = \frac{3\alpha_+}{4M_T^2}$.

Next, the auxiliary fields H, L_i, and R_i are introduced and the new action for ψ_t has the form:

$$
S_{4,t} = \int \{-(\bar{\psi}_{t,L}^a H_{ab}^+ \psi_R^b + (h.c.)) - \frac{4M_T^2}{3\alpha_+} H_{ab}^+ H_{ab}\} d^4x
$$
$$
+ \int \{(\bar{\psi}_{t,L}^a \gamma^i L_i^{ab} \psi_{t,L}^b) - \frac{4M_T^2}{3\alpha_+} \mathrm{Tr}\, L^i L_i + (L \longleftrightarrow R)\} d^4x \quad (31)
$$

Integrating out fermion fields we arrive at the effective action for the mentioned auxiliary fields (and the source currents for fermion bilinears). The resulting effective action receives its minimum at $H = m_t \mathbf{1}$, where m_t plays the role of the technifermion mass (equal for all technifermions).

We apply the following regularization:

$$
\frac{1}{p^2 + m^2} \to \int_{\frac{1}{\Lambda_\chi^2}}^\infty d\tau e^{-\tau(p^2 + m^2)} \qquad (32)
$$

With this regularization the expression for the condensate of ψ_t is (after

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the Wick rotation):

$$
\begin{aligned}
< \bar{\psi}_t \psi_t > \;=\;& N_t \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p\gamma + m} = -N_t m_t \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + m_t^2} \\
=\;& -N_t m_t \int_{\frac{1}{\Lambda_\chi^2}}^{\infty} d\tau \int \frac{d^4 p}{(2\pi)^4} e^{-\tau(p^2 + m_t^2)} \\
=\;& -\frac{N_t}{16\pi^2} 4 m_t^3 \Gamma(-1, \frac{m_t^2}{\Lambda_\chi^2})
\end{aligned}
\tag{33}
$$

Here $\Gamma(n, x) = \int_x^\infty \frac{dz}{z} e^{-z} z^n$. The gap equation is:

$$
m_t = -g_s < \bar{\psi}_t \psi_t >
\tag{34}
$$

That is

$$
m_t = G_S m_t \{ \exp(-\frac{m_t^2}{\Lambda_\chi^2}) - \frac{m_t^2}{\Lambda_\chi^2} \Gamma(0, \frac{m_t^2}{\Lambda_\chi^2}) \}
\tag{35}
$$

It does not depend on G_V. Obviously, there exists the critical value of G_S: at $G_S > 1$ the gap equation has the nonzero solution for m_t while for $G_S < 1$ it has not. This means that in this approximation the condensation of technifermions occurs at

$$
M_T < M_T^{\text{critical}} = \sqrt{3\alpha_+ N_t} \frac{\Lambda_\chi}{4\pi} \sim \sqrt{\alpha_+} \Lambda_\chi
\tag{36}
$$

For example, at $\Lambda_\chi \sim 10$ Tev and $\alpha_+ \sim 1/100$ we may have $M_T^{\text{critical}} \sim 1$ Tev.

The technipion decay constant F_T in the given approximation is:

$$
F_t = \frac{N_t m_t^2}{4\pi^2} \Gamma(0, \frac{m_t^2}{\Lambda_\chi^2})
\tag{37}
$$

Therefore,

$$
F_t^2 = \frac{N_t \Lambda_\chi^2}{4\pi^2} e^{-\frac{m_t^2(M_T, \Lambda_\chi)}{\Lambda_\chi^2}} - \frac{4 M_T^2}{3\alpha_+}
\tag{38}
$$

In order to have appropriate values of W and Z - boson masses we need $F_T \sim 250$ Gev. At $M_T = M_T^{\text{critical}}$ we have $m_t = 0$ and $F_T = 0$. When M_T is decreased, m_t increases and reaches the value around Λ_χ somewhere at $M_T = M_T^{\text{critical}}/2$. At this point $F_T \sim \frac{\sqrt{N_t} \Lambda_\chi}{4\pi}$. As $\Lambda_\chi > 1$ Tev we need $\frac{M_T^{\text{critical}} - M_T}{M_T^{\text{critical}}} <<$ 1. Usual naturalness requirement here means that $x = \frac{[M_T^{\text{critical}}]^2 - M_T^2}{[M_T^{\text{critical}}]^2} \sim 0.1$.

Smaller values of this ratio would be considered unnatural. At small x we have: $F_T \sim \sqrt{N_t/2 \frac{\Lambda_\chi}{2\pi}} x \sim 0.25$ Tev. Thus naturalness forbids to consider extremely large values of Λ_χ (say, of the order of Plank mass). That's why we bound ourselves by the values of Λ_χ between 1 Tev and 10 Tev.

Negative G_V leads to the appearance of the term in the action with $(\rho_L^2 + \rho_R^2)$, where $\rho_L^{ab} = (\bar{\psi}_{t,L}^a \gamma^0 \psi_{t,L}^b)$ and $\rho_R^{ab} = (\bar{\psi}_{R,L}^a \gamma^0 \psi_{R,L}^b)$ are the densities of right-handed and left - handed technifermions. This is an attractive interaction that qualitatively corresponds to a positive shift of the chemical potential μ. That's why negative G_V moves chiral symmetry restoration to smaller values of μ. However, we expect this change at $G_V = -\frac{1}{4} G_S$ does not affect physics at $\mu = 0$ although this is to be the subject of an additional investigation.

In the absence of SM interactions the relative orientation of the SM gauge group $G_W = SU(3) \otimes SU(2) \otimes U(1)$ and $SU(\mathcal{N})_V$ from $SU(\mathcal{N})_L \otimes SU(\mathcal{N})_R \to SU(\mathcal{N})_V$ is irrelevant. However, when the SM interactions are turned on, the effective potential due to exchange by SM gauge bosons depends on this relative orientation. Minimum of the potential is achieved in the true vacuum state and defines the pattern of the breakdown of G_W. This process is known as vacuum alignment (see, for example, [34, 35]). The effective potential is [34]:

$$
\begin{aligned}
V(U) &= 4 \sum_{\alpha = SU(3), SU(2), U(1); \, k} e_\alpha^2 \, \mathrm{Tr}\, (\theta_L^{\alpha,k} U \theta_R^{\alpha,k} U^+) \\
&\quad (-\frac{i}{2}) \int d^4 x \Delta^{\mu\nu}(x) < 0 | T[J_{\mu L}^A J_{\nu R}^A | 0 > \\
&= -\frac{3}{32\pi^2} (F^2 \Delta^2) \sum_{\alpha = SU(3), SU(2), U(1); \, k} e_\alpha^2 \, \mathrm{Tr}\, (\theta_L^{\alpha,k} U \theta_R^{\alpha,k} U^+) \quad (39)
\end{aligned}
$$

There is no sum over A here. $\theta_{L,R}^{\alpha,k}$ are generators of G_W, $\Delta^{\mu\nu}(x)$ is the gauge boson propagator, $J_{\mu L;R}^A = (\bar{\psi}_{t,L;R}^a \lambda_{ab}^A \gamma_i \psi_{t,L;R}^b)$ are technifermion currents; matrices λ_{ab}^A are generators of $SU(\mathcal{N})$. $U \in SU(\mathcal{N})$ defines relative orientation of $SU(\mathcal{N})_V$ and G_W. F - is the technipion constant. In general case Δ^2 may be negative. However, in [34] arguments are given in favor of $\Delta^2 > 0$. Namely, it was shown that if the technicolor interactions are renormalizable and asymptotic free, then the spectral function sum rules take place. Then under assumption that in the spectral functions correspondent to vector and axial vector channels of $< 0 | T[J_{\mu L}^A J_{\nu R}^A | 0 >$ single intermediate states dominate, one finds $\Delta^2 > 0$. In our case dynamical torsion plays the role of

the technicolor interactions. That's why we need some suppositions about the dynamical torsion theory. In particular, if we require that this theory is renormalizable and asymptotic free (as it should in order to be self - consistent) and that two intermediate states dominate in the mentioned above correlator, we also have $\Delta^2 > 0$. Under this supposition in a way similar to that of [34] we come to the conclusion that G_W is broken in a minimal way. This means that the subgroups of G_W are not broken unless they should. The form of the condensate requires that $SU(2)$ and $U(1)$ subgroups are broken. That's why in a complete analogy with $SU(N_{TC})$ Farhi - Susskind model Electroweak group in our case is broken correctly while $SU(3)$ group remains unbroken.

7   Mass term

In this section we consider the possibility to give masses to the SM fermions. Namely, let us consider the action with an additional mass term for spinors ψ and ϕ:

$$
\begin{aligned}
S_f &= \int \{ \frac{i}{2}\bar{\psi}_a\gamma^\mu(\zeta - i\chi\gamma^5)D_\mu\psi_a + (c.c.)\}d^4x \\
&+ \int \{ \frac{i}{2}\bar{\phi}^c{}_b\gamma^\mu(\zeta - i\chi\gamma^5)\bar{D}_\mu\phi^c_b + (c.c.)\}d^4x \\
&- \int (\delta_{aa'}\bar{\psi}_a\psi_{a'} + \mathbf{f}_{bb'}\bar{\phi}_b\phi_{b'})m_0 d^4x
\end{aligned}
\tag{40}
$$

Here m_0 is the constant of the dimension of mass while $\mathbf{f}$ is the hermitian matrix of couplings. Integrating over torsion we obtain:

$$
\begin{aligned}
S_f &= \int \{i\bar{\psi}^a_s\gamma^\mu D_\mu\psi^a_s - \frac{3\alpha_-}{16M_T^2}(\bar{\psi}^a_s\gamma^i\gamma^5\psi^a_s)(\bar{\psi}^b_s\gamma_i\gamma^5\psi^b_s)\}d^4x \\
&+ \int \{i\bar{\psi}^a_t\gamma^\mu D_\mu\psi^a_t + \frac{3\alpha_+}{16M_T^2}(\bar{\psi}^a_t\gamma^i\gamma^5\psi^a_t)(\bar{\psi}^b_t\gamma_i\gamma^5\psi^b_t)\}d^4x \\
&- \frac{m_0}{\sqrt{[\mathrm{Re}\zeta]^2 - [\mathrm{Im}\chi]^2}} \int (\bar{\psi}_{s,a}[\frac{\delta_{aa'} - \mathbf{f}_{aa'}}{2} - \gamma^5\frac{\delta_{aa'} + \mathbf{f}_{aa'}}{2}]\psi_{t,a'} + (c.c.))d^4x
\end{aligned}
\tag{41}
$$

Here we have composed new spinors $\psi^a_t = \sqrt{[\mathrm{Re}\zeta] - [\mathrm{Im}\chi]}\begin{pmatrix} \phi^a_- \\ \psi^a_+ \end{pmatrix}$ and

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$$\psi_s^a = \gamma^5\sqrt{[\mathrm{Re}\zeta] + [\mathrm{Im}\chi]}\begin{pmatrix} \psi_-^a \\ \phi_+^a \end{pmatrix}.$$

Next, we neglect SM gauge fields that are to be considered as perturbations. We also introduce the auxiliary fields as in the ENJL approach:

$$
\begin{aligned}
S_f &= \int\{i\bar\psi_s^a\gamma^\mu D_\mu\psi_s^a - \frac{3\alpha_-}{16M_T^2}(\bar\psi_s^a\gamma^i\gamma^5\psi_s^a)(\bar\psi_s^b\gamma_i\gamma^5\psi_s^b)\}d^4x \\
&+ \int\{i\bar\psi_t^a\gamma^\mu D_\mu\psi_t^a\}d^4x \\
&+ \int\{-(\bar\psi_{t,L}^a H_{ab}^+\psi_{t,R}^b + (h.c.)) - \frac{4M_T^2}{3\alpha_+}\mathrm{Tr}\,H^+H\}d^4x \\
&+ \int\{(\bar\psi_{t,L}^a\gamma^i L_i^{ab}\psi_{t,L}^b) - \frac{4M_T^2}{3\alpha_+}\mathrm{Tr}\,L^iL_i + (L\longleftrightarrow R)\}d^4x \\
&- \frac{m_0}{\sqrt{[\mathrm{Re}\zeta]^2 - [\mathrm{Im}\chi]^2}}\int(\bar\psi_{s,a}[\frac{\delta_{aa'} - \mathbf{f}_{aa'}}{2} - \gamma^5\frac{\delta_{aa'} + \mathbf{f}_{aa'}}{2}]\psi_{t,a'} + (c.c.))d^4x
\end{aligned}
$$
(42)

Integration over technifermions leads to appearance of the effective potential for H that has its minimum at $H = m_t\mathbf{1}$. So, $H = m_t\mathbf{1} + h$, where vacuum value of h is zero. Thus we get:

$$
S_f = \int\{i\bar\psi_s^a\gamma^\mu D_\mu\psi_s^a - \frac{3\alpha_-}{16M_T^2}(\bar\psi_s^a\gamma^i\gamma^5\psi_s^a)(\bar\psi_s^b\gamma_i\gamma^5\psi_s^b)\}d^4x + S_{eff}[L,R,H]
$$
$$
- \frac{m_0^2}{[\mathrm{Re}\zeta]^2 - [\mathrm{Im}\chi]^2}\int\{\begin{pmatrix}\psi_{s,L} \\ -\mathbf{f}\psi_{s,R}\end{pmatrix}^+\gamma^0[i\gamma^\mu D_\mu - m_t\mathbf{1} - h]^{-1}\begin{pmatrix}\psi_{s,L} \\ -\mathbf{f}\psi_{s,R}\end{pmatrix}\}d^4x
$$
(43)

where $D_\mu = (\partial_\mu - i\frac{1+\gamma_5}{2}L_\mu - i\frac{1-\gamma_5}{2}R_\mu)$. Here we denote $S_{eff} = -i\mathrm{Sp}\,\mathrm{Log}[i\gamma^\mu D_\mu - m_t\mathbf{1} - h]$.

Now our supposition is that $m_t >> m_0$. Next, at the energies much less than M_T we can omit the four fermion terms for ψ_s. We also neglect fluctuations of h, L, and R around their zero vacuum values and arrive at:

$$
S_f = \int\bar\psi_s(i\gamma^\mu\partial_\mu - \frac{m_0^2}{[\mathrm{Re}\zeta]^2 - [\mathrm{Im}\chi]^2}\mathbf{f}[m_t]^{-1})\psi_s d^4x \qquad (44)
$$

As a result the mass term for ψ_s appears with the mass matrix

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$$m_s = \frac{m_0^2}{[\mathrm{Re}\zeta]^2 - [\mathrm{Im}\chi]^2} \frac{\mathbf{f}}{m_t} \qquad (45)$$

It is worth mentioning that in order to have m_s positive defined we need $[\mathrm{Re}\zeta]^2 > [\mathrm{Im}\chi]^2$ provided that $\mathbf{f}$ is also positive defined. When $[\mathrm{Re}\zeta]^2 < [\mathrm{Im}\chi]^2$ we can compose ψ_s as follows: $\psi_s^a = \sqrt{[\mathrm{Re}\zeta] + [\mathrm{Im}\chi]} \begin{pmatrix} \psi_-^a \\ \phi_+^a \end{pmatrix}$ and arrive at $m_s = \frac{m_0^2}{[\mathrm{Im}\chi]^2 - [\mathrm{Re}\zeta]^2} \frac{\mathbf{f}}{m_t}$.

8 Conclusions

In this paper we considered fermions coupled in a nonminimal way to Poincare gravity. The gravity action contains the Holst action with the mass parameter at a Tev scale. In addition the fermion action itself breaks Parity. That's why the left - right asymmetry appears in the effective four - fermion interactions. We arrange all SM fermions in the left - handed components of the Dirac spinors while the right - handed components are reserved for technifermions. Via an appropriate choice of couplings the four - fermion terms that contain SM fermions can be made repulsive while the four - fermion terms that contain technifermions can be made attractive. We also need that the four fermion interaction that contains both SM fermions and technifermions is negligible (23). Therefore, the technifermions are condensed and cause the appearance of W and Z - boson masses. We need $\frac{M_T}{\sqrt{\alpha_+}} < \Lambda_\chi$, where Λ_χ is the scale, at which the dynamical Poincare gravity theory appears. We expect $M_T \sim 1$ Tev. At the same time the scale Λ_χ is expected to be between 1 Tev and 10 Tev.

An obvious difficulty of our approach is that we need the effective coupling constants to satisfy condition (23). That's why the detailed analysis of the renormalization group trajectories is needed in order to investigate the necessary domain of the theory. We also need to know what does it mean: $|\alpha_{+-}| << \alpha_+$. For example, is this sufficient or not to have $|\alpha_{+-}| \sim 0.1\,\alpha_+$, may become clear only after the detailed analysis of the NJL model is performed. Namely, we need to investigate the NJL model that includes ψ_s and ψ_t with the four fermion term that includes both ψ_t and ψ_s.

In order to provide appearance of masses for the SM fermions we add the mass term for the Dirac spinors that contain SM fermions as their left-handed components. This term is considered as a perturbation over the four

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- fermion interactions caused by torsion. As a result the mass term for the SM fermions appears.

There is the important question about the scale of M_T, Λ_χ, and the mass parameter entering (40) that gives rise to SM mass matrix. Actually, if one assumes that quantum gravity theory (that is the dynamical theory of metric) exists at the energies of the order of Planck mass m_p, such mass parameters might be generated dynamically and, therefore, receive values at a m_p scale. Therefore we must suppose that there exists a mechanism within the m_p scale theory that forbids dynamical generation of torsion mass as well as m_0 from (40). Actually, we may suppose that there is no quantum theory of Riemannian geometry at all. Then the dynamical torsion theory may be thought of as a gauge theory of Lorentz group that is defined in Minkowsky space [36, 37]. This theory may have a scale slightly above 1 TeV. In this approach there is no problem with the scale m_p at all. In such a scheme classical gravity may appear, for example, as an entropic force [38].

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